Wave interference with an obstacle

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Introduction

High school students learning physics will at some point hear of the slit experiment, where monochromatic laser light of wavelength λ goes through a single slit of width d. λ and d should be of the same order of magnitude.

As a result, an interference pattern will appear on a screen behind the slit, which might look like this:

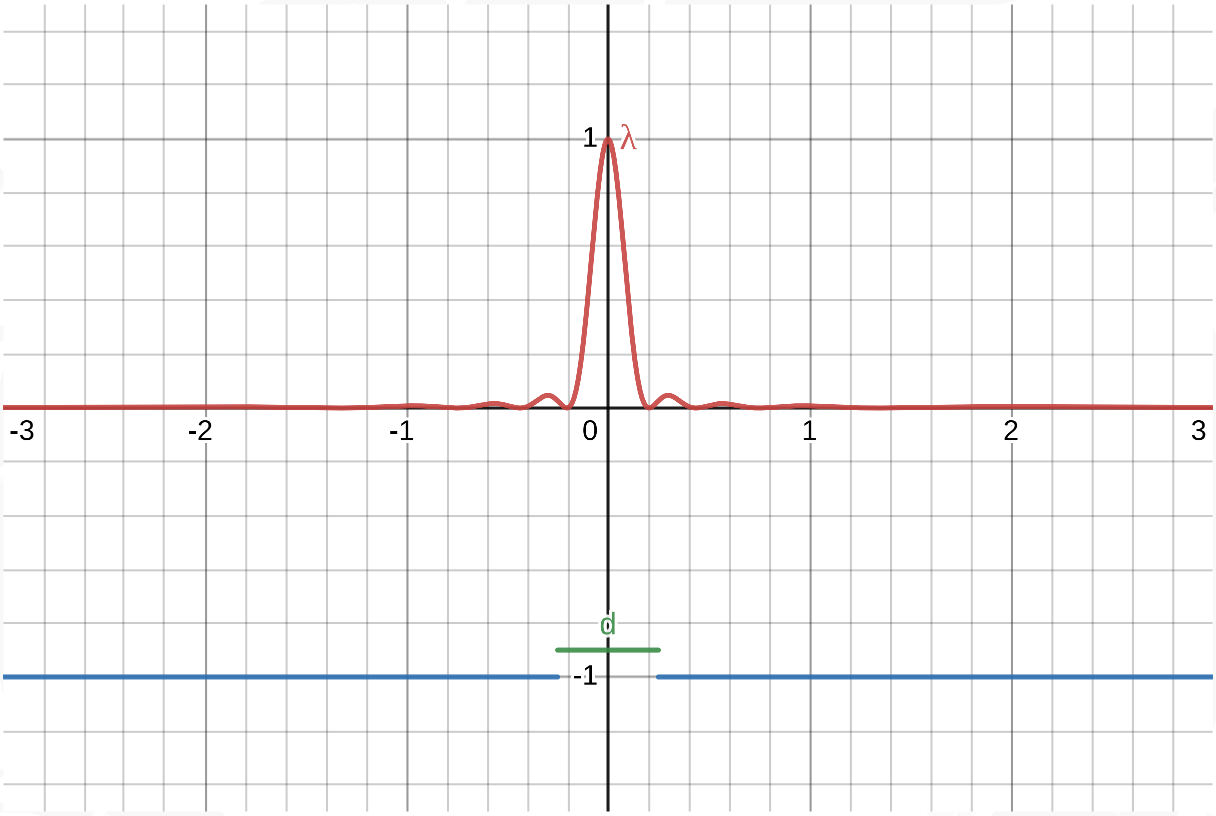

Figure 1: Single-slit interference pattern.

A variant of the single slit experiment would be interference at an obstacle instead of a slit, which shall also have a width of d. In practice, this might be a single hair, for example. In much the same way that the slit width d can be determined from the interference pattern, this can also be achieved for the obstacle. In fact, the minima in the interference patterns for both cases are in the same locations as will be derived, here.

This is a short one showing that the interference pattern of an obstacle is the same as the pattern of a single slit with the same width. (If the observation angle is sufficiently large.)

For the single slit, the locations of the minima in Fraunhofer-approximation are

$$d\,\sin\theta_n = n\lambda,$$

where θ_n is the angle that corresponds to the nth minimum.

Derivation

In order to understand why the minima locations (or angles) are identical for a slit and an obstacle of width d, consider three systems.

Firstly, there is the regular slit, indicated by s. Secondly, there is the obstacle system which will be indicated by o.

Lastly, consider a completely free system, indicated by f, where there is nothing obstructing the light at all.

The light's amplitude at a given angle θ in system i will be denoted as $A_i(\theta)$.

By the principle of superposition,

$$A_f(\theta) = A_s(\theta) + A_o(\theta).$$

Assuming the light ray's radius (corresponding to θ_r) is smaller than the position of the first minimum on the screen, i. e. the screen is sufficiently far away from the slit or obstacle, the amplitude in the free system will be zero:

$$A_f(\theta \geq \theta_r) = 0$$

Thus,

$$A_o(\theta \geq \theta_r) = -A_s(\theta \geq \theta_r).$$

Now, there is one final thing. On the screen, it is not the amplitudes but the *intensities* $I_i \propto A_i^2$ of the light that make up the interference pattern. Therefore, the last equation must be squared:

$$I_o(\theta \geq \theta_r) \propto I_s(\theta \geq \theta_r)$$

Therefore, the interference patterns (for sufficiently large screen distances) are indeed identical.